

Calculus Summer Assignment

Due: First day of Class

I (Mr. Houck) am available for help throughout the summer. Just e-mail me, houck@longleafschool.com. I will be traveling from the end of June throughout most of July. So, e-mails might sit for a day or two before I respond to them. Do not let that deter you from e-mailing me.

This assignment will be in two parts. Part 1 is a series of problems I want you to do. Each one in itself is very short, but there are enough that this will take some time. For example, I consider problem 1 to have 36 parts. Each of those 36 questions just takes a couple of steps, but there are a lot of problems. (I did one question as an example; so there are only 35 questions you need to answer.)

Each one of these problems will challenge your algebra and problem-solving skills. In Calculus, the students who are overwhelmed with the course are the ones who take the "easy" or "quick" path when they struggle. By that I mean that they look up solutions on the internet or claim they work with others when they in fact just copy down what another student does. The quick path keeps them from growing, and dooms them to overwhelming difficulties later on. The ones who succeed are the ones who fight through struggles and grow throughout the course so that when the really hard math comes, they can find success. So, I want you to work on these problems completely on your own, and just check with me if you want to verify that you have correct answers.

Please take advantage of my offer for assistance. Many students who take Calculus have never needed to ask their math teacher for help before. It is a difficult thing to do that the first time. I won't give you the answer, but I will walk alongside you as you overcome each frustration and obstacle. Taking on Calculus is like going to a gym and lifting an amount of weight you have never before attempted. I am here to give a little boost, if needed. But, you have to lift most of the weight yourself, because this is how you grow to lift the much heavier weights that await us later on in the course.

I promise I will not cause you to feel unworthy or unprepared if you ask a question. I will not internally classify you as a second-rate student. Instead, I will think and communicate that you are a brilliant Longleaf student who had a hard time seeing the right key that unlocks the problem.

Part 2 is an essay. It asks for your definition of what it means to be "good at math," and then asks if you are good at math. I will grade it based on how honest you are in your writing. Do not write what you think I want to read. Instead, write about your own journey with math. Include successes (if any) and struggles (if any.) The problems in Part 1 will be a part of your math journey. You are welcome to talk about your encounter with them or not. Taking math classes remotely, as we did from March through May, is also a part of your math journey. You are welcome to talk about that experience or not. There's a lot that goes into one's definition of being good at math, and my criteria is honesty. Please write honestly and you will earn full credit for your essay.

Part 1

1. Define the functions, f_1, \dots, f_6 as follows: $f_1(x) = x$, $f_2(x) = \frac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \frac{1}{1-x}$, $f_5(x) = \frac{x-1}{x}$, and $f_6(x) = \frac{x}{x-1}$

Fill out the composition table. Simplify after composing, and show your work below. Read the table as row composed with column. For example,

$$f_3 \circ f_4 = 1 - \left(\frac{1}{1-x} \right) = \frac{1-x}{1-x} - \frac{1}{1-x} = \frac{-x}{1-x} = f_6.$$

You must simplify after composing. (*Hint: This collection of functions is a closed collection. This means that the composition of any two of the functions above gives another one in the list after you simplify the composition.*)

| \circ | f_1 | f_2 | f_3 | f_4 | f_5 | f_6 |
|---------|-------|-------|-------|-------|-------|-------|
| f_1 | | | | | | |
| f_2 | | | | | | |
| f_3 | | | | f_6 | | |
| f_4 | | | | | | |
| f_5 | | | | | | |
| f_6 | | | | | | |

Show work below.

2. The sum and difference formulas for $\sin(x)$ and $\cos(x)$ are as follows:

$$\sin(a \pm b) = \sin(a) \cos(b) \pm \sin(b) \cos(a)$$

$$\cos(a \pm b) = \cos(a) \cos(b) \mp \sin(a) \sin(b)$$

Use these to show Double Angle Formulas are true.

(a) $\sin(2x) = 2 \sin(x) \cos(x)$

(b) $\cos(2x) = \cos^2(x) - \sin^2(x)$

(c) $\cos(2x) = 2 \cos^2(x) - 1$

(d) $\cos(2x) = 1 - 2 \sin^2(x)$

3. Use the Double Angle Formulas to show the Half-angle Formulas are true.

(a) $\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos(x)}{2}}$

(b) $\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos(x)}{2}}$

4. Let $f(x)$ and $g(x)$ be given by the following table.

| | | | | | | |
|--------|----|----|---|----|---|---|
| x | -3 | -2 | 0 | 1 | 3 | 5 |
| $f(x)$ | 2 | -2 | 3 | 0 | 0 | 4 |
| $g(x)$ | 1 | -3 | 0 | -2 | 5 | 3 |

Find the following. If the result cannot be determined, write DNE.

(a) $f(0)$

(b) $g^{-1}(-2)$

(c) $f \circ g(3)$

(d) $g \circ f(-3)$

(e) $f \circ g(0)$

(f) $g \circ f(0)$

(g) $g^{-1} \circ f(-2)$

(h) $f^{-1} \circ g(5)$

(i) $g^{-1} \circ f(5)$

(j) $f^{-1} \circ g(0)$

5. $\sinh(x)$, the hyperbolic $\sin(x)$ function, is defined as $\sinh(x) = \frac{e^x - e^{-x}}{2}$. Similarly, the hyperbolic $\cos(x)$, $\cosh(x) = \frac{e^x + e^{-x}}{2}$.

(a) Show that $\cosh^2(x) - \sinh^2(x) = 1$.

(b) Show that $\sinh(x) \cosh(y) + \sinh(y) \cosh(x) = \sinh(x + y)$

(c) Show that, for any real number r , $(\cosh(x) + \sinh(x))^r = \cosh(rx) + \sinh(rx)$.

(d) For those who've seen Calculus before.

i. Define $\tanh(x)$ in the most reasonable way. Find $\frac{d}{dx}(\sinh(x))$, $\frac{d}{dx}(\cosh(x))$, and $\frac{d}{dx}(\tanh(x))$.
Simplify to get similar answers to $\frac{d}{dx}(\sin(x))$ etc...

ii. Find $\int \sinh(x) dx$, and $\int \cosh(x) dx$.

Part 2

Write a 500 word essay (approximately one page) that addresses the following topic: Briefly describe what it means to be "good at math." According to your definition, are you good at math? Support your answer with examples. (You may type your essay in the space below, or provide it in a separate document.)